Product Quality and Consumer Search *

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Abstract

An increase in quality shifts up the distribution of match utilities offered by firms and makes consumers pickier. The number of products consumers inspect does not necessarily increase in quality. Higher search costs may lead to less quality investment and the equilibrium price may decrease. If the equilibrium is inefficient, it is because of the inadequacy of quality investment. The market level of quality investment is excessive (insufficient) and consumers are too (little) picky from the point of view of welfare maximization if and only if a rise in quality results in consumers inspecting a higher (lower) number of products.

Keywords: quality investment, consumer search, efficiency.

JEL Classification: D43, D83, L13

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In most markets, even if they are Internet-based, consumers have to incur significant search costs to inspect products. While the consumer search literature has paid a lot of attention to the influence of search costs on the intensity of search and firm pricing (see e.g. Wolinsky, 1986; and Anderson and Renault, 1999), little is known about the interaction between product quality, consumer search and prices.[1] This paper performs a positive and normative study of the provision of quality in a consumer search market. We investigate a version of Wolinsky's (1986) work-horse model in which infinitely many firms selling differentiated products compete by setting prices and investing in quality, while consumers search sequentially until they find a satisfactory product. A rise in quality increases the entire distribution of match utilities offered by a firm (in the sense of first-order stochastic dominance) and therefore consumers who pay it a visit are more likely to be satisfied, stop search and buy its product. Investing in quality is of course costly for the firms, which generates a trade-off for them. In this setting, we ask three main questions.

- First, how is consumer search affected by an increase in quality? Do consumers check more or less products when they face "better" distributions of match utilities?
- How do search frictions influence the market equilibrium? Do higher search costs lead to higher or lower quality? How are prices affected by an increase in search costs?
- Do firms provide the socially optimal amount of quality? Under which conditions can we expect an over- or under-provision of quality?

Let us start with the question whether consumers inspect more or less products after an increase in quality. Consumer optimal search is characterized by a stationary reservation value that increases in firms' investment in quality. Hence, consumers become more "choosy" as quality increases. That consumers become more picky as quality rises, however, does not necessarily imply that they inspect more products. This is because the direct effect of a rise in quality is to lower the number of products consumers check, as when quality is higher they find a suitable product faster. Either of these two effects may dominate. For example, an increase in quality results in consumers checking more products when match values are distributed according to the Uniform I, Exponential I, and Kumaraswamy I distributions (see Table 1 below); the opposite occurs for the Uniform II, Exponential II and the Kumaraswamy II distributions. More generally, the direct effect dominates the choosiness effect and hence consumers check fewer products after an increase in quality when the expected difference between the valuation of a consumer who finds an acceptable product and the reservation value decreases in quality. We relate this intuition to the notion of (decreasing) *mean residual life* ordering of distribution functions.

A candidate market equilibrium is a stopping rule for the consumers and a price-investment decision for the firms that are consistent with one another. We provide sufficient conditions under which a candidate market equilibrium exists and is unique. These conditions are easy to

[1] For a recent, authoritative, survey of firm behavior with consumer search, see Anderson and Renault (2018).

verify and we provide a variety of examples, including some based on uniform and exponential distributions, for which a candidate market equilibrium exists, is unique and may easily be characterized. For the candidate market equilibrium to actually be an equilibrium, the payoff function of an individual firm must be (quasi-)concave both in price and quality investment. When the density of match values is increasing and the cost of investment sufficiently convex, the equilibrium exists.

How do search frictions influence quality investment? The relationship between competition and investment has preoccupied economists at least since Schumpeter (1950) and Arrow (1962). Though the existing literature on this theme is extensive (see Vives, 2008), as far as we know, the relationship between competition and investment has not been examined in frictional markets. When we study how quality investment depends on search costs, which is the natural measure of competitiveness in our setting, we find that the relationship is quite intricate because a firm's incentive to invest in quality depends on three quantities that vary with search costs, namely, its margin per buyer, the typical number of visitors of the firm and the marginal increase in the probability consumers stop searching at its premises. A higher search cost increases the margin, reduces the number of visitors and may increase or decrease the marginal rise in the stopping probability.

To the best of our knowledge, Ershov (2018) is the only empirical paper measuring the effect of lower search costs on quality. Using data from the Google Play store, Ershov argues that a re-categorization of part of the store that took place in 2014 reduced search costs for game apps and shows empirically that this led to lower quality apps. Our model provides a result consistent with Ershov's finding for some match utility distributions. However, we also show that there exist other distributions for which the opposite holds.

Regarding the question how search costs affect the equilibrium price, we first note that, as in Wolinsky (1986, 2005), harsher search frictions are associated with higher prices (see e.g. Anderson and Renault, 1999). However, when quality is endogenous, the equilibrium price not only depends on the reservation value of consumers but also on quality investment. Because investment in quality may increase or decrease in search costs, the overall impact of higher search costs on the equilibrium price is generally ambiguous. If an increase in quality raises the measure of marginal matches more than the measure of supramarginal matches, then demand elasticity increases and the price falls. Therefore, for distributions satisfying the increasing hazard-ratio ordering (Shaked and Shanthikumar, 2007), *ceteris paribus*, higher quality translates into higher prices. We then conclude that when search costs go up and quality too as a result, the equilibrium price increases provided that the distribution of match values satisfies the increasing hazard-ratio ordering. This occurs for the case of the the Uniform II and more generally Kumaraswamy II distributions (see Table 1). Otherwise, when higher search costs lead to lower quality being supplied in the market, the equilibrium price may decrease. To illustrate this latter possibility, we provide an example based on the exponential distribution in which both the equilibrium price and quality investment decrease as search costs go up.

We finally turn to the question whether the market provides the socially optimal amount of quality. Social welfare equals the expected value of the match utility minus the total incurred costs, which include the search costs and the costs of investment in quality. We first observe that, conditional on a given quality investment, consumer search is socially optimal. Because the equilibrium price does not matter for social welfare, this implies that the only source of inefficiency in the search market stems from the inadequacy of the incentives to invest in quality. We show the existence of a one-to-one relationship between the intensity of search and the inefficiency of the market equilibrium. When a rise in quality results in consumers inspecting a higher number of products, then the market level of quality investment is insufficient from the point of view of welfare maximization. In contrast, the market level of quality is excessive when a rise in quality decreases the number of products consumers check before buying.

Related literature

The main contribution of this paper is to perform a positive and normative study of the welfare aspects of quality provision in a competitive environment with search costs. In a seminal contribution, Spence (1975) showed that a monopolist may under- or over-invest compared to the social planner. Because search costs lead to monopolistic competition (Wolinsky, 1986), it is natural that some of the insights of Spence resonate in our setting. For example, like in Spence's article, we also obtain that the market may provide excessive or insufficient quality depending on how quality shifts the demand function. However, a crucial distinction is that while the monopoly price is pivotal to the determination of the efficiency of quality investment, in our model with frictions the equilibrium price does not matter and what really plays a role is the reservation value of consumers. Interestingly, we show a novel relationship between the efficiency of the market provision of quality and the (expected) number of products consumers inspect before stopping search.

As far as we know, work on the normative analysis of quality provision in consumer search markets is scant.[2] The first paper is by Wolinsky (2005). There are two important differences between Wolinsky's setup and ours. The first is that he assumes that an individual firm has to invest in quality every time a new consumer pays a visit to the firm. His model thus better applies to sellers that invest in *service* quality and therefore the nature of quality is short-run in his setup. In our model, by contrast, an individual seller invests once and for all in quality, which corresponds to the more traditional view of quality as a long-run decision variable.

[2]However, there is a related stream of early work that was mainly motivated by the question whether professionals should be allowed to advertise prices and, in particular, how this price advertising would affect quality provision. Chan and Leland (1982) study a model with all-or-nothing search in which firms are Stackelberg leaders vis-á-vis consumers. As in Salop and Stiglitz (1977), there are two types of consumers and in market equilibrium there is either one or two price-quality combinations. They show that allowing for price advertising improves consumer welfare. They extend their model to the case of sequential search in Chan and Leland (1986) and derive conditions under which the same result arises. Rogerson (1988) assumes a sequential search protocol and heterogeneous consumers both in search costs and marginal des-utility of price. He derives an equilibrium with a continuum of price-quality combinations and shows that allowing for price advertising, which activates the role of prices as signals of quality, is shown to improve consumer welfare. A contrasting result is derived by Dranove and Satterthwaite (1992) in a model in which prices and qualities are observed with noise. Random variables are normally distributed and the equilibrium quality turns out to be independent of search costs.

Given a quality level, the short-run nature of the investment implies that consumers search too much from the point of view of social welfare, while in our model of long-run investment consumer search is efficient. The second main difference is that in Wolinsky's model quality enters additively in the consumers' utility function. We show in our paper that the additive case is special in that neither demand nor the equilibrium price vary with the investment and the market provision of quality turns out to be efficient in our setting. Moreover, quality is independent of search costs.

The second paper is by Fishman and Levy (2015). They present a related model in which firms either sell a product of high or of low quality. In their model quality is a long-run variable and, like in Wolinsky (2005), enters the match value distribution additively. They show that if the share of high-quality firms is initially low, then higher search costs result in more investment in quality; otherwise, as frictions go up, quality decreases. Fishman and Levy do not study the efficiency of the market equilibrium.[3]

A third, more recent, paper studying the efficiency of *service* provision is Janssen and Ke (2020). Their focus is on markets where service value is fully transferable across firms. In equilibrium, at least two firms offer service and therefore, because service is a public good, service provision is excessive from a welfare point of view.

Finally, Chen, Li and Zhang (2019) also study the optimality of quality provision in a consumer search market. The main difference between their paper and ours is that they consider the case of experience goods, that is, products whose quality can only be ascertained after consumption. They find that equilibrium investment in product quality is insufficient (excessive) when search cost is low (high).[4]

More generally, our paper adds to a growing literature allowing for a richer choice of firm strategies in consumer search models. In particular, in Bar-Isaac, Caruana and Cuñat (2012) vertically and horizontally differentiated firms choose price and design for their products. Following Johnson and Myatt (2006) in that different product designs induce demand rotations, they show that low-quality firms choose niche, only appealing to a few, designs while high-quality firms go for broad, appealing to most, designs. Larson (2013) studies the efficiency of product designs in a similar model to that in Bar-Isaac, Caruana and Cuñat (2012) where product designs induce mean-preserving spreads of the match value distribution. He shows that when search costs are low, the market choose maximally dispersed utility distributions and this is efficient. Finally, in Gamp (2019) consumers have heterogeneous (uniformly distributed)

[3]Two recent papers build on the framework of Fishman and Levy (2015). Gamp and Krähmer (2019) studies firms' incentives to invest in quality when some consumers are naive in the sense that they neither observe nor infer product characteristics. In such an environment, lowering search costs pushes firms to focus on the naive consumers and quality falls. Moraga-González and Sun (2019) studies the incentives of firms to provide service quality in a model in which quality enters the utility function multiplicatively and, like in Wolinsky (2005), firms have to put effort every time they get a new consumer at their premises. They show that higher search costs result in higher quality.

[4]A few papers in the consumer search literature have studied other efficiency aspects of the market equilibrium. Anderson and Renault (1999) study the efficiency of entry; they find that the market equilibrium always provides an excessive number of firms. In a related paper, Chen and Zhang (2015) finds that entry is excessive from a welfare viewpoint when the costs of entering the market are low enough, while for high entry costs entry is deficient for consumer welfare.

search costs and are allowed to purchase without inspecting the products. He shows that in such a setting all firms target niches in equilibrium. Our model is related to these papers in that investments in quality induce demand shifts; these demand shifts may in general increase or decrease the elasticity of demand, which turns out to be important for the price effects of quality investments.[5]

The paper is organized as follows. We introduce the model in section 1. Section 2 is dedicated to the characterisation of the symmetric equilibrium. In this section, we also provide conditions under which a market equilibrium exists and is unique. Section 3 examines the effect of higher search costs on the equilibrium price and quality investment. Section 4 presents the characterisation of the social optimum. In this section, we also provide conditions under which the market over- or under-supplies quality. Finally, section 5 provides some concluding remarks. To ease the reading, all the proofs are relegated to an Appendix. Our working paper Moraga-González and Sun (2022) provides a couple of illustrative examples fully solved.

1 Model

The market has a unit mass of consumers and a unit mass of sellers. In the tradition started by Wolinsky (1986) and followed up by many recent contributions (see the Introduction section), firms sell horizontally differentiated products and consumers visit them sequentially in order to find a satisfactory match. The novelty of our model is that firms can make costly efforts to improve their products; we refer to this effort as *investment in quality.*

Consumers are initially imperfectly informed about their fit with the products of the firms and their prices, but they discover these features by visiting them. We refer to this activity as *search.* Each time a consumer visits a firm, she incurs a search cost, denoted by c. The purpose of search is to inspect the products of the firms, see how well they suit the consumer and find out at which price they sell. While searching, consumers hold at all times correct (passive) beliefs about the equilibrium investment in quality and the equilibrium price.[6]

Let p_i be the price charged by firm i. A consumer m who buys product i gets a utility equal to

$$u_i^m = \varepsilon_i^m - p_i,$$

where ε_i^m represents the value of the match between consumer m and product i. We assume that the match values offered by a firm i are distributed according to a continuous and twice differentiable distribution function $F(\varepsilon;\lambda_i)$, with density $f(\varepsilon;\lambda_i)$ and support $[\underline{\varepsilon}(\lambda_i), \overline{\varepsilon}(\lambda_i)]$.

[5] Other papers have considered alternative strategies such as advertising and merging. Haan and Moraga-González (2011) present a consumer search model where firms gain prominence by investing in persuasive advertising. Armstrong and Zhou (2011), Choi, Dai and Kim (2018) and Haan, Moraga-González and Petrikaitė (2018) present models in which firms' prices are advertised. Finally, Moraga-González and Petrikaitė (2013) study firms' incentives to merge and the aggregate implications of mergers. Rhodes and Zhou (2019) also study firms' incentives to merge and retail various products but in a setting where consumers buy multiple products.

[6] As Bar-Isaac, Caruana and Cuñat (2012) and Larson (2013) argue, in the absence of common factors influencing firms' decisions, with (infinitely many) firms that pick price and quality independently, it is reasonable to expect that a consumer cannot infer much about the deals available at other firms upon observing a deviation at one of the firms.

The match values are independent, both across consumers and across firms. As it is common in the literature, we assume that $1 - F(\varepsilon; \lambda_i)$ is log-concave in ε.

The variable $\lambda_i \in [0, \overline{\lambda}]$ is a choice of firm i and represents firm i's investment in quality: an increase in λ_i increases the match value distribution in the sense of *first-order stochastic dominance* (FOSD).[7] Put it in mathematical terms, we assume that

$$\frac{\partial(1 - F(\varepsilon; \lambda_i))}{\partial \lambda_i} \geq 0 \tag{1}$$

and finite for all ε (and strictly positive for some ε's). The lower and upper bounds of the support of ε may or may not depend on λ_i.

Investment in quality is costly. Let $K(\lambda_i)$ represent the cost of investing λ_i; we assume that the function K is twice differentiable, increasing and sufficiently convex. Sufficient convexity of K reflects the presumption that it is significantly increasingly costly to provide better products to the population of consumers. This assumption, which we make precise later, ensures that the firms' pricing and investment problem is well-behaved. The case of $\lambda_i = 0$ represents the baseline case of no investment. We assume that $K(0) = K'(0) = 0$. The other extreme case if that of $\lambda_i = \overline{\lambda}$, which represents maximal investment. To avoid corner solutions in investment, we assume that $K'(\overline{\lambda}) = \infty$, so the optimal investment of a firm will always be less than $\overline{\lambda}$. To ensure that the equilibrium price is also interior we let $c \in [\underline{c}, \overline{c}]$, with $\underline{c} > 0$, and assume that $\underline{\varepsilon}(\overline{\lambda})$ is sufficiently low.[8]

Interaction in the market is as follows. Firms simultaneously choose their prices and quality investments to maximize their profits. The marginal cost of production is normalised to zero. Then, without observing prices, investments and match utilities, consumers search sequentially in the market until they find a satisfactory product. We focus on symmetric pure-strategy equilibrium, that is, an equilibrium in which all firms charge the same price and make the same investment to improve their products. For the market to exist, search costs must be low enough so that consumers find it worthwhile to search.[9]

Before moving to the equilibrium analysis, we mention two special cases of our demand formulation that have received attention in the literature. The first instance is the *additive* case (cf. Wolinsky, 2005) where an investment in quality λ_i increases consumer m's gross utility from buying product i from ε_i^m to $\lambda_i + \varepsilon_i^m$. In our formulation, this example is captured by the distribution of match values $F(\varepsilon - \lambda_i)$. In the additive case, the utility of a consumer increases by the same amount no matter the match value. The second instance is the *multiplicative* case (cf. Anderson and Renault, 2000) where an investment in quality λ_i increases consumer m's

[7] As suggested by an anonymous reviewer, λ can also be interpreted as a measure of advertising persuasiveness.

[8] We assume the search cost is bounded away from zero because in our model when search costs equal zero the equilibrium price is also zero, in which case no firm would engage in any costly investment in quality. That $\underline{\varepsilon}(\overline{\lambda})$ is sufficiently low is a technical condition that we specify later in Section 2.

[9] For this, it suffices that $c \leq \overline{c} \equiv \min_{\lambda \in [0,\overline{\lambda}]} \left\{ \int_{p^m(\lambda)} (\varepsilon - p^m(\lambda)) f(\varepsilon; \lambda) d\varepsilon \right\}$, where $p^m(\lambda)$ stands for the monopoly price corresponding to the distribution of match values $F(\varepsilon; \lambda)$, i.e. $p^m(\lambda) \equiv \arg\max_p p(1 - F(p; \lambda))$. For some of the distributions in Table 1 below, $\overline{c}$ can be computed explicitly. For example, for the Uniform I, $\overline{c} = \frac{1+\lambda}{8}$; for the Uniform II, $\overline{c} = \frac{1}{8(1-\lambda)}$; for the Exponential I, $\overline{c} = \frac{1+\lambda}{e}$; and for the Exponential II, $\overline{c} = e^{1+\lambda}$. For the other distributions, it is not possible because the monopoly price cannot be computed in closed-form.

gross utility from product i from ε_i^m to $(1+\lambda_i)\varepsilon_i^m$. In our formulation, this is captured by the distribution of match values $F(\frac{\varepsilon}{1+\lambda_i})$. In the multiplicative case, consumers with high initial match values benefit more from an increase in quality than the others. We shall return to these special cases later.

In order to illustrate the results of our paper, we use the following families of distributions of match values.

Name	**CDF**	**MRL (wrt λ)**	**HR (wrt λ)**	**Price interior**	**Unique eq. candidate**	**Existence of eq.**
Uniform I	$F(\varepsilon;\lambda)=\frac{\varepsilon}{1+\lambda}$, $\varepsilon\in[0,1+\lambda]$, $\lambda>0$	incr.	decr.	✓	✓	K'' large
Uniform II	$F(\varepsilon;\lambda)=\frac{\varepsilon-\lambda}{1-\lambda}$, $\varepsilon\in[\lambda,1]$, $0\le\lambda<1$	const.	const.	$\overline{\lambda}$ small	K'' large	K'' large
Power	$F(\varepsilon;\lambda)=\left(\frac{\varepsilon}{1+\lambda}\right)^{\alpha}$, $\varepsilon\in[0,1+\lambda]$, $\alpha>1$, $\lambda>0$	incr.	decr.	✓	✓	K'' large
Exponential I	$F(\varepsilon;\lambda)=1-e^{-\frac{\varepsilon}{1+\lambda}}$, $\varepsilon\in[0,\infty]$, $\lambda>0$	incr.	decr.	✓	✓	K'' large
Exponential II	$F(\varepsilon;\lambda)=1-e^{-(\varepsilon-\lambda)}$, $\varepsilon\in[\lambda,\infty]$, $\lambda>0$	const.	const.	$\overline{\lambda}$ small	✓	K'' large
Kumaraswamy I	$F(\varepsilon;\lambda)=1-\left(1-\frac{\varepsilon}{1+\lambda}\right)^{\alpha}$, $\varepsilon\in[0,1+\lambda]$, $\alpha>0$, $\lambda>0$	incr.	decr.	✓	✓	K'' large
Kumaraswamy II	$F(\varepsilon;\lambda)=1-\left(1-\frac{\varepsilon-\lambda}{1-\lambda}\right)^{\alpha}$, $\varepsilon\in[\lambda,1]$, $\alpha>0$, $0\le\lambda<1$	const.	const.	$\overline{\lambda}$ small	K'' large	K'' large

Table 1: Illustrative distribution functions

The Kumaraswamy distribution is often used as a substitute for the beta distribution (see Ding and Wolfstetter, 2011). All the distributions in Table 1 have log-concave failure functions. Moreover, for all of them an increase in λ signifies an increase in the match value distribution in the FOSD sense. The first two columns on the right hand side of the table report whether the distributions have *mean residual life* functions and *hazard rates* increasing or decreasing in λ, which we define later. The last columns on the right hand side of the table indicate, respectively, conditions for which the following four statements hold: the price equilibrium is interior (cf. Section 2.2), a unique candidate equilibrium exists (cf. Lemma 1), a market equilibrium exists and is unique (cf. Proposition 3). The "✓" symbol means that the statement holds without additional assumptions. The sufficient conditions for these statements to hold appear later in the paper. We will return to this table when discussing them.

2 Market equilibrium

In this section, we study the existence and uniqueness of a *symmetric* equilibrium of the model. In a symmetric equilibrium all sellers choose the same quality level and charge the same price; as a result, the utility distribution is the same across all sellers. Moreover, all consumers search in the same way, holding correct conjectures about prices and match utility distributions. We start with the characterisation of optimal consumer search.

2.1 Consumer search and the number of searches

Let (p,λ) be consumers' expectation about the firm equilibrium. Because all firms are expected to offer the same utility distribution, we can rely on Kohn and Shavell (1974), who show that the optimal search rule for a consumer who faces a set of independently and identically distributed options with a known distribution is static in nature and has the stationary reservation utility property.

Accordingly, consider the expression

$$\int_{\varepsilon}^{\overline{\varepsilon}(\lambda)} (z-\varepsilon) f(z;\lambda) dz. \tag{2}$$

Equation (2) is the (equilibrium) marginal benefit of search to a consumer who has a match value ε at hand, which is clearly a decreasing function of ε. The consumer continues to inspect products so long as (2) is greater than the search cost c. Correspondingly, we define the reservation value $\hat{\varepsilon}$ as the unique solution in ε to:

$$h(\varepsilon;\lambda) \equiv \int_{\varepsilon}^{\overline{\varepsilon}(\lambda)} (1-F(z;\lambda)) dz = c, \tag{3}$$

where we have rewritten (2) using integration by parts. The function h is monotonically decreasing in ε. Moreover, $h(\underline{\varepsilon}(\lambda);\lambda) = \mathbb{E}[\varepsilon;\lambda]$ and $h(\overline{\varepsilon}(\lambda);\lambda) = 0$. Therefore, for a fixed λ, there exists a unique interior reservation value $\hat{\varepsilon}$ for any $c \in [\underline{c}, \mathbb{E}[\varepsilon;\lambda]]$. Because $\mathbb{E}[\varepsilon;\lambda]$ increases in λ, and since $c \leq \overline{c} < \mathbb{E}[\varepsilon;0]$, a unique interior reservation value exists for any $\lambda \in [0, \overline{\lambda}]$.

Notice that the reservation value $\hat{\varepsilon}$ is a function of the search cost c and firm investment in quality λ. The standard comparative statics results hold. Applying the implicit function theorem to equation (3) gives

$$\frac{\partial \hat{\varepsilon}}{\partial c} = -\frac{1}{1-F(\hat{\varepsilon},\lambda)} < 0. \tag{4}$$

Therefore, for a fixed λ, as it is well-known, the reservation value $\hat{\varepsilon}$ decreases in the search cost c, revealing that consumers' inclination to check products decreases as the search cost goes up.

Likewise,

$$\frac{\partial \hat{\varepsilon}}{\partial \lambda} = \frac{\int_{\hat{\varepsilon}}^{\overline{\varepsilon}(\lambda)} \frac{\partial(1-F(z;\lambda))}{\partial \lambda} dz}{1-F(\hat{\varepsilon},\lambda)} > 0, \tag{5}$$

which implies that, for a given search cost c, $\hat{\varepsilon}$ is an increasing function of λ.[10] This means that when the market supplies better products in the sense of FOSD, consumers become more choosy and, hence, are more willing to continue to inspect products in the market until they find a satisfactory match. Equation (5) also reveals that it is only the change in the right tail of the distribution of ε's above the initial reservation value $\hat{\varepsilon}$ that is relevant for the impact of an increase in quality on consumers' reservation value.

That consumers become more choosy as quality increases does not necessarily mean that they check more products before they stop search. In fact, the number of products checked by consumers may increase or decrease in quality investment. This depends on the nature of the FOSD shift of the distribution of match values caused by an increase in quality. To see this, note that the equilibrium number of searches of a typical consumer is equal to

$$S(\hat{\varepsilon};\lambda) \equiv \frac{1}{1-F(\hat{\varepsilon};\lambda)}. \tag{6}$$

[10]Kohn and Shavell (1974) mention this relationship for the case in which quality enters utility additively. Weitzmann (1979) does so more generally.

Taking the derivative of the number of searches with respect to quality investment gives:

$$\begin{aligned}\frac{dS}{d\lambda} &= \frac{\partial S}{\partial \lambda} + \frac{\partial S}{\partial \hat{\varepsilon}}\frac{\partial \hat{\varepsilon}}{\partial \lambda} \\ &= \frac{1}{(1-F(\hat{\varepsilon};\lambda))^2}\left[\underbrace{-\frac{\partial(1-F(\hat{\varepsilon};\lambda))}{\partial \lambda}}_{\text{direct effect }(-)} + \underbrace{\frac{f(\hat{\varepsilon};\lambda)}{1-F(\hat{\varepsilon};\lambda)}\int_{\hat{\varepsilon}}^{\overline{\varepsilon}(\lambda)}\frac{\partial(1-F(z;\lambda))}{\partial \lambda}dz}_{\text{“choosiness” effect }(+)}\right],\end{aligned} \tag{7}$$

where we have used (5). On the one hand, this expression reveals that the “direct effect” of higher quality is to reduce the number of searches because the chance that a product is good enough for them goes up in quality. On the other hand, there is a “choosiness effect” because the reservation value is endogenous and higher quality makes consumers pickier. By this effect the number of searches tends to increase because consumers are more likely to discard products while they search for a satisfactory one. Thus:

Proposition 1 *A small increase in quality results in consumers inspecting a higher (lower) number of products if and only if:*

$$\frac{\frac{\partial(1-F(\hat{\varepsilon};\lambda))}{\partial\lambda}}{f(\hat{\varepsilon};\lambda)} < (>) \frac{\int_{\hat{\varepsilon}}\frac{\partial(1-F(z;\lambda))}{\partial\lambda}dz}{1-F(\hat{\varepsilon};\lambda)}. \tag{8}$$

Moreover, when quality enters additively, the number of searches is constant in quality and when quality enters multiplicatively, the number of searches increases in quality.

Proof. See the Appendix. ■

When condition (8) holds, the choosiness effect is more powerful than the direct effect and therefore the number of searches increases in quality investment. The LHS of (8) represents the increase in the stopping probability caused by an increase in quality per marginal consumer. The RHS is the change in the reservation value caused by an increase in quality. Condition (8) then says that consumers will inspect more products after a small quality increase when the increase in the stopping probability per marginal consumer is lower than the increase in the reservation value itself. Intuitively, consumers will inspect more products when a rise in quality increases the upper tail of the distribution of match values more than other lower parts of the distribution.

Condition (8) is a necessary and sufficient condition for the number of searches to increase (decrease) in quality. Moreover, it can readily be checked for a given distribution of match values. For example, an increase in quality results in consumers checking more products when match values are distributed according to the Uniform I, Exponential I, and Kumaraswamy I; the opposite occurs for the Uniform II, Exponential II and the Kumaraswamy II cases.

Although condition (8) is easy to check for a given distribution of match values, it may be useful to provide more general families of distributions for which it is satisfied. For this it is convenient to introduce the notion of *mean-residual-life* (MRL) of the random variable ε as well as the MRL stochastic ordering of random variables. The MRL of the random variable ε with distribution function $F(\varepsilon, \lambda)$ is:

$$MRL(x; \lambda) = \mathbb{E}[\varepsilon - x | \varepsilon > x] = \frac{\int_x^{\overline{\varepsilon}(\lambda)} \varepsilon f(\varepsilon, \lambda) d\varepsilon}{1 - F(x, \lambda)} - x, \text{ for all } x < \overline{\varepsilon}(\lambda).$$

The MRL function is used much in actuarial sciences and industrial engineering and relates to the upper tail of the distribution of match values.[11] Evaluated at the reservation value, $MRL(\hat{\varepsilon}; \lambda)$ gives the difference between the expected valuation of a consumer who finds an acceptable product and the reservation value. A random variable ε with log-concave failure function $1 - F(\varepsilon; \lambda)$ has a MRL decreasing in x (Bagnoli and Bergstrom, 2005). This holds for all the distributions in Table 1. The family of distributions $F(\varepsilon; \lambda)$ is said to satisfy the *decreasing (increasing) MRL ordering* if and only if $MRL(x; \lambda)$ weakly decreases (increases) in λ.

Proposition 2 *If the random variable ε satisfies the decreasing MRL ordering, then a small increase in quality results in consumers inspecting a (weakly) lower number of products.*

Proof. See the Appendix. ■

A sufficient condition for the number of searches to decrease in quality investment is that the match value distribution satisfies the decreasing MRL ordering. In such a case, the difference between the expected valuation of a consumer who finds an acceptable product and the reservation value decreases in the reservation value and in quality. Hence, consumers will search less after an increase in quality. The Uniform II, Kumaraswamy II and Exponential II families of distributions are examples of distributions satisfying the decreasing MRL ordering. The three families have a MRL that is constant in λ. Another example, not listed in Table 1, is the family of distribution functions $F(\varepsilon, \lambda) = \frac{(\varepsilon-\lambda)(1-\lambda\varepsilon)}{(1-\lambda)^2}$, $0 \leq \lambda < 1$, $\varepsilon \in [\lambda, 1]$, which has a MRL decreasing in λ and in x.

Other sufficient conditions have been put forward in contemporaneous work. Choi and Smith (2020) show that consumers search more (fewer) products when the match value satisfies the *increasing (decreasing) dispersive ordering* while Zhou (2020) does so when the match value satisfies the *increasing (decreasing) excess wealth ordering.* Both the dispersive ordering and the condition in our Proposition 2 imply the excess wealth ordering.

2.2 Firm pricing and investment

Let $\hat{\varepsilon} \in (\underline{\varepsilon}(\lambda), \overline{\varepsilon}(\lambda))$ be firms' expectation about consumers' search behaviour. Assume that $\min_{\lambda \in [0, \overline{\lambda}]} \left\{ \frac{1-F(\hat{\varepsilon};\lambda)}{f(\hat{\varepsilon};\lambda)} \right\} > \underline{\varepsilon}(\overline{\lambda})$, which ensures that the equilibrium price (to be computed) is interior.[12] We now characterise an interior pure-strategy symmetric equilibrium in price

[11] Kotz, S. and D. N. Shanbhag (1980) show that knowledge of the MRL of a random variable uniquely identifies its distribution function. We thank an anonymous reviewer for making us aware of this result.

[12] This condition holds for the Uniform I, Power, Exponential I, and Kumaraswamy I families of distributions. For the Uniform II, Exponential II, and Kumaraswamy II families, it holds when $\overline{\lambda}$ is sufficiently small (cf. Table 1).

and investment, denoted (p, λ). To do this, we write out the payoff to a firm, say i, that deviates from the proposed equilibrium by charging a price p_i and investing an amount λ_i, taking as given other firms' equilibrium decisions and consumers' search behaviour. After this, we compute the first order conditions (FOCs), apply symmetry and study the existence of a symmetric (price and investment) equilibrium.

The payoff to a deviant firm charging a price $p_i \neq p$, and investing an amount $\lambda_i \neq \lambda$ is equal to:

$$\pi_i(p_i, \lambda_i; p, \lambda) = S(\hat{\varepsilon}; \lambda) p_i [1 - F(\hat{\varepsilon} - p + p_i; \lambda_i)] - K(\lambda_i). \tag{9}$$

This expression should be understood as follows. The per buyer revenue of the deviant firm is p_i; the number of consumers who visit the deviant firm is $S(\hat{\varepsilon}; \lambda)$; the probability that one of these visitors chooses to stop searching and buys the product of the deviant firm is the probability of the event $\varepsilon_i - p_i \geq \hat{\varepsilon} - p$, which equals $1 - F(\hat{\varepsilon} - p + p_i; \lambda_i)$. The costs of investing λ_i enter the profit expression negatively.

Taking the first-order derivatives of the profit function with respect to λ_i and p_i and applying symmetry ($p_i = p$, $\lambda_i = \lambda$) gives the following FOCs:

$$p = \frac{1 - F(\hat{\varepsilon}; \lambda)}{f(\hat{\varepsilon}; \lambda)} \tag{10}$$

$$p \frac{\frac{\partial (1 - F(\hat{\varepsilon}; \lambda))}{\partial \lambda}}{1 - F(\hat{\varepsilon}; \lambda)} - K'(\lambda) = 0. \tag{11}$$

Plugging p into the second expression, we can rewrite the FOCs (10)-(11) as follows:

$$p = \frac{1 - F(\hat{\varepsilon}, \lambda)}{f(\hat{\varepsilon}, \lambda)} \tag{12}$$

$$0 = \frac{\frac{\partial (1 - F(\hat{\varepsilon}, \lambda))}{\partial \lambda}}{f(\hat{\varepsilon}, \lambda)} - K'(\lambda). \tag{13}$$

For a given $\hat{\varepsilon}$, if a symmetric firm equilibrium exists, all sellers choose p and λ satisfying (12) and (13) as their optimal strategy. Inspection of this system of equations shows that it is "triangular": the equilibrium λ follows directly from (13) as a function of the consumers' reservation value; once λ is computed, the equilibrium price p follows from the corresponding inverse hazard rate formula usual in monopoly and monopolistic competition settings (cf. Wolinsky, 1986).

We now provide conditions under which a symmetric firm equilibrium candidate exists and is unique.

Lemma 1 *For a fixed reservation value $\hat{\varepsilon}$, if $\frac{\partial (1 - F(\hat{\varepsilon}, 0))}{\partial \lambda} > 0$, a symmetric firm equilibrium candidate that solves (p, λ) exists. If, in addition, $\frac{1}{f^2} \left(\frac{\partial^2 (1 - F)}{\partial \lambda^2} f - \frac{\partial (1 - F)}{\partial \lambda} \frac{\partial f}{\partial \lambda} \right) - K''(\lambda) < 0$, then the symmetric firm equilibrium candidate is unique.*

Proof. See the Appendix. ■

Note that the assumption in Lemma 1 that $\frac{\partial (1 - F(\hat{\varepsilon}, 0))}{\partial \lambda} > 0$ is innocuous when investment shifts the entire distribution of match values downwards, which is the usual case in most of the examples of the paper. The condition for unicity is satisfied when investment cost is sufficiently convex but this is not necessary in many cases. For example, when match values follow the Uniform I, Power, Kumaraswamy I, and Exponential I and II families of distributions any convex cost function will do. Moreover, when match values follow the Uniform II and the Kumaraswamy II, the convex cost function $K(\lambda) = \left(\frac{\lambda}{1-\lambda}\right)^2$ suffices.

The existence of a symmetric firm equilibrium candidate (p, λ) that solves (12) and (13) does not necessarily imply that it is indeed an equilibrium. To ensure this, we shall assume that the payoff function in (9) is strictly concave in both p_i and λ_i. The exact conditions necessary for this are given in the Appendix where we analyze the Hessian matrix corresponding to the payoff function in (9). The conditions suggest that when demand is concave in p_i (for which an increasing density of match values suffices) and the cost of investment function is sufficiently convex, a symmetric firm equilibrium candidate is indeed a firm equilibrium. For all the distributions in Table 1, a sufficiently convex investment cost function suffices for the existence of equilibrium. Note that the necessary degree of convexity may potentially be higher than that required for the existence of a unique symmetric firm equilibrium candidate (as in the cases of the Uniform II and Kumaraswamy II distributions). In what follows, we shall assume that a symmetric firm equilibrium exists and is unique.

2.3 Market equilibrium

In the above two subsections, we have seen how consumers search optimally when they expect a price p and a level of investment λ; likewise, we have seen how firms choose their prices and quality investments when they expect consumers to search using a reservation value $\hat{\varepsilon}$. A market equilibrium exists if there is a triplet $(\hat{\varepsilon}^*, p^*, \lambda^*)$ that simultaneously solves the consumers' and the sellers' problems. Because the price p^* is uniquely pinned down from (12) after plugging $\hat{\varepsilon}^*$ and λ^*, a market equilibrium exists if there is a pair $(\hat{\varepsilon}^*, \lambda^*)$ that solves the following system of equations:

$$\int_{\hat{\varepsilon}} (1 - F(z; \lambda))dz - c = 0, \tag{14}$$

$$\frac{\frac{\partial(1-F(\hat{\varepsilon},\lambda))}{\partial\lambda}}{f(\hat{\varepsilon}, \lambda)} - K'(\lambda) = 0. \tag{15}$$

This system combines the consumers' search rule (14) and the sellers' profit maximization problem (15).[13]

Equation (14) defines an implicit relation between the reservation value and investment, which, as we have discussed in Section 2.1, is monotonically increasing. Let $\hat{\varepsilon}_1(\lambda)$ denote such a relationship and notice that $\underline{\varepsilon}(0) < \hat{\varepsilon}_1(0) < \overline{\varepsilon}(0)$ and $\underline{\varepsilon}(\overline{\lambda}) < \hat{\varepsilon}_1(\overline{\lambda}) < \overline{\varepsilon}(\overline{\lambda})$. Equation (15) also defines implicitly a relationship between the reservation value and investment. Let $\hat{\varepsilon}_2(\lambda)$ denote this second relationship. A symmetric market equilibrium $(\hat{\varepsilon}^*, p^*, \lambda^*)$ exists if $\hat{\varepsilon}_1(\lambda)$ and

[13] Because now the reservation value $\hat{\varepsilon}$ is endogenous, the equilibrium price is interior provided that $\min_{\lambda\in[0,\overline{\lambda}]}\left\{\frac{1-F(\hat{\varepsilon}(\lambda);\lambda)}{f(\hat{\varepsilon}(\lambda);\lambda)}\right\} > \underline{\varepsilon}(\overline{\lambda})$, where $\hat{\varepsilon}(\lambda)$ solves (14).

$\hat{\varepsilon}_2(\lambda)$ cross one another. The market equilibrium is unique if they cross each other a single time.

Applying the implicit function theorem to (15) gives:

$$\frac{\partial \hat{\varepsilon}_2}{\partial \lambda} = \frac{\frac{1}{f^2}\left(\frac{\partial^2(1-F)}{\partial \lambda^2} f - \frac{\partial(1-F)}{\partial \lambda}\frac{\partial f}{\partial \lambda}\right) - K''(\lambda)}{\frac{1}{f^2}\left(\frac{\partial f}{\partial \lambda} f + \frac{\partial(1-F)}{\partial \lambda}\frac{\partial f}{\partial \hat{\varepsilon}}\right)}.$$

Note that under the conditions in Lemma 1 the numerator of this expression is negative. However the sign of the denominator is ambiguous. In fact, depending on the distributional assumption for the match values, it may be positive or negative, which means that $\hat{\varepsilon}_2(\lambda)$ may be increasing or decreasing. For example, $\hat{\varepsilon}_2(\lambda)$ is increasing for the Uniform I, Exponential I, Power, and Kumaraswamy I, while it is decreasing for the Uniform II, Exponential II, Kumaraswamy II and for the distribution $F(\varepsilon, \lambda) = \frac{(\varepsilon-\lambda)(1-\lambda\varepsilon)}{(1-\lambda)^2}$, $0 \leq \lambda < 1$, $\varepsilon \in [\lambda, 1]$. An increasing (decreasing) relationship means that firms' incentives to invest increase if firms believe consumers are more (less) choosy.

Our next result provides sufficient conditions under which a market equilibrium exists and is unique. We provide two sets of conditions, depending on whether (15) defines an increasing or a decreasing relationship between the reservation value and investment.

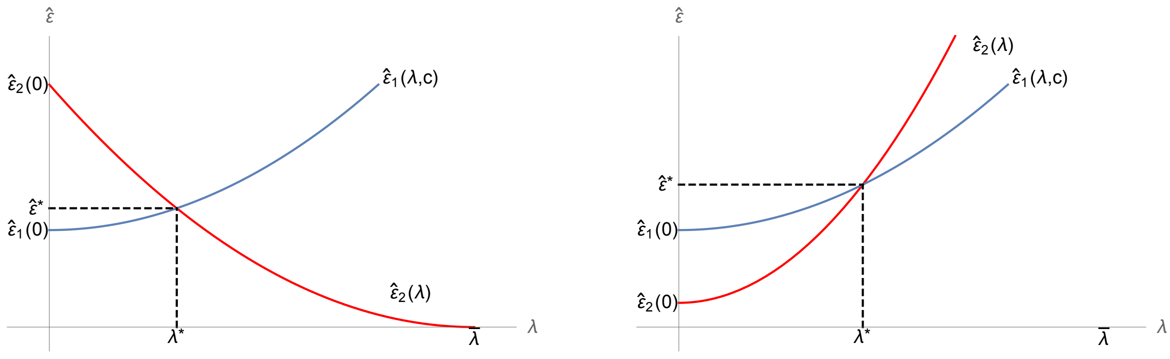
Proposition 3 *Assume that the conditions in Lemma 1 hold and that the payoff function in (9) is strictly concave in both* p_i *and* λ_i*. Then, a unique market equilibrium exists if, in addition, either of the following conditions hold:*

(a) $\frac{\partial f}{\partial \lambda} f + \frac{\partial(1-F)}{\partial \lambda}\frac{\partial f}{\partial \varepsilon} \geq 0$, $\hat{\varepsilon}_2(0) > \hat{\varepsilon}_1(0)$ *and* $\hat{\varepsilon}_1(\overline{\lambda}) > \hat{\varepsilon}_2(\overline{\lambda})$,

(b) $\frac{\partial f}{\partial \lambda} f + \frac{\partial(1-F)}{\partial \lambda}\frac{\partial f}{\partial \varepsilon} < 0$, $\hat{\varepsilon}_1(0) > \hat{\varepsilon}_2(0)$ *and* $\hat{\varepsilon}_2(\overline{\lambda}) > \hat{\varepsilon}_1(\overline{\lambda})$,

Consumers' reservation value $\hat{\varepsilon}^*$ *and firms quality investment* λ^* *solve (14) and (15). The equilibrium price* p^* *is given by (12).*

This Proposition is illustrated in Figure 1. The conditions on the values the functions $\hat{\varepsilon}_1(\lambda)$ and $\hat{\varepsilon}_2(\lambda)$ take on the boundaries of the set $[0, \overline{\lambda}]$ ensure that the market equilibrium exists and is unique. These conditions hold for all the distributions in Table 1. Therefore, for all the distributions in Table 1 either (a) or (b) hold so Proposition 3 does not impose any additional requirement. In the working paper version of our paper we fully develop the characterization of the market equilibrium for two illustrative examples, namely, the cases of the Uniform II and the Exponential I.



(a) $\frac{\partial \hat{\varepsilon}_2}{\partial \lambda} < 0$ (b) $\frac{\partial \hat{\varepsilon}_2}{\partial \lambda} > 0$

Figure 1: Existence and uniqueness of equilibrium

3 Higher search costs

The relationship between the incentives to invest (both in process and product innovations) and market power has attracted a lot of interest in economics at least since Schumpeter (1950) and Arrow (1962). In this literature, typical measures of competitiveness have been the number of firms or the degree of product differentiation (see e.g. Vives, 2008). In consumer search models, the cost of search naturally lends itself as a measure of competition. We now ask how the market equilibrium, and in particular quality investment, is affected by higher search costs. Interestingly, investment can increase or decrease, and the equilibrium price, in contrast to the conventional result in the literature, need not increase in search cost.

3.1 The effect of higher search costs on equilibrium investment

A careful look at equation (11) reveals that the private gains from investing in quality are proportional to the equilibrium price, the usual number of visitors of a firm, and the change in the probability they stop at the premises of the firm:

$$S(\hat{\varepsilon}^*;\lambda^*)p^*\frac{\partial(1-F(\hat{\varepsilon}^*,\lambda^*))}{\partial\lambda}. \tag{16}$$

After using the formula for the equilibrium price given in (12), we can rewrite the expression in (16) as:

$$\frac{\frac{\partial(1-F(\hat{\varepsilon}^*,\lambda^*))}{\partial\lambda}}{f(\hat{\varepsilon}^*,\lambda^*)}. \tag{17}$$

The impact of higher search costs on the incentives to invest in quality are thus governed by the sensitivity of the incremental rise in the acceptance probability per marginal consumer with respect to the search cost:

$$\frac{\partial}{\partial c}\left(\frac{\frac{\partial(1-F(\hat{\varepsilon}^*,\lambda^*))}{\partial\lambda}}{f(\hat{\varepsilon}^*,\lambda^*)}\right) = -\frac{1}{[f(\hat{\varepsilon}^*,\lambda^*)]^2}\left(\frac{\partial f(\hat{\varepsilon}^*,\lambda^*)}{\partial\lambda}f(\hat{\varepsilon}^*,\lambda^*) + \frac{\partial(1-F(\hat{\varepsilon}^*,\lambda^*))}{\partial\lambda}\frac{\partial f(\hat{\varepsilon}^*,\lambda^*)}{\partial\hat{\varepsilon}}\right)\frac{\partial\hat{\varepsilon}^*}{\partial c}.$$

Because $\partial\hat{\varepsilon}^*/\partial c < 0$, we can use this derivative and Proposition 3 to directly provide sufficient conditions under which an increase in search costs results in higher or lower quality investment.

Proposition 4 *(a) If the conditions in Proposition 3(a) hold, then, as the search cost rises, the reservation value $\hat{\varepsilon}$ decreases and the equilibrium investment level λ^* increases.*

(b) If the conditions in Proposition 3(b) hold, then, as the search cost increases, both the reservation value $\hat{\varepsilon}$ and the investment level λ^* decrease.

An increase in search costs gives consumers incentives to accept products that are less satisfactory, which is reflected in a decrease in consumers' reservation value. When the match value distribution satisfies the conditions in Proposition 3(a) (e.g. for the Uniform II, Exponential II and Kumaraswamy II cases) the marginal gains from an investment in quality increase in consumers' reservation value and therefore higher search costs result in a higher quality investment. Alternatively, when the conditions in Proposition 3(b) hold (e.g. for the Uniform I, Exponential I, Power and Kumaraswamy I) and the fall in the consumers' reservation value makes the increase in the acceptance probability bigger and the number of consumers at the margin lower, higher search costs result in a lower quality investment.

3.2 The effect of higher search costs on equilibrium price

The impact of higher search costs on the equilibrium price is quite complex because there are two effects at work and these effects need not go in the same direction. The first effect is due to the impact of higher search costs on consumers' reservation value, while the second effect is due to the impact of higher search costs on investment.

In order to make some progress, it is useful to introduce the *hazard rate (HR) ordering* of random variables. The hazard rate corresponding to the distribution of match values is given by:

$$HR(\varepsilon;\lambda) = \frac{f(\varepsilon,\lambda)}{1 - F(\varepsilon,\lambda)}.$$

We say that the family of distributions $F(\varepsilon,\lambda)$ satisfies the *increasing (decreasing) hazard rate ordering* if and only if $HR(\varepsilon;\lambda)$ weakly decreases (increases) in λ. The increasing (decreasing) HR ordering implies the FOSD (reverse FOSD) ordering, not vice versa (see Theorem 1.B.1 in Shaked and Shanthikumar, 2007). Likewise, the HR ordering implies the MRL ordering (see Theorem 2.A.1 in Shaked and Shanthikumar, 2007). When the distribution of match values satisfies the increasing (decreasing) HR ordering, *ceteris paribus*, the demand of a firm becomes less (more) elastic so a higher quality translates into a higher (lower) price.

The following derivative gives the effect of higher search costs on the equilibrium price:

$$\frac{dp^*}{dc} = \underbrace{\frac{\partial p^*}{\partial \hat{\varepsilon}}}_{\leq 0}\underbrace{\frac{\partial \hat{\varepsilon}}{\partial c}}_{<0} + \underbrace{\frac{\partial p^*}{\partial \lambda^*}}_{?}\underbrace{\frac{\partial \lambda^*}{\partial c}}_{?}. \tag{18}$$

On the one hand, higher search costs result in a lower reservation value, which increases the equilibrium price (cf. Wolinsky, 1986; Anderson and Renault, 1999). On the other hand, higher search costs may result in a lower or higher investment level (cf. Proposition 4). Whether this effect results in a higher price depends on how the hazard rate varies with λ. Our next result, for which we do not need a detailed proof, provides a sign to the ambiguous derivatives in (18). In addition, it gives the impact of search costs for various examples of distributions given in Table 1.

Proposition 5 *If the conditions in Proposition 3(a) (3(b)) hold and ε satisfies the increasing (decreasing) HR ordering, then the equilibrium price increases in search cost. Moreover:*

(a) When match values follow the Uniform II and more generally the Kumaraswamy II, the equilibrium price increases in search costs.

(b) When match values follow the Exponential I distribution, the equilibrium price decreases in search costs.

(c) When match values follow the Exponential II distribution, the equilibrium price does not vary in search costs.

The different results put forward by this Proposition have to do with the different signs the derivatives in (18) can take. We now explain why the Proposition holds. If the conditions in Proposition 3(a) hold, we have $\partial\lambda^*/\partial c > 0$. If, in addition, $\partial p^*/\partial\lambda^* > 0$, the total derivative in (18) will be positive; this occurs when the distribution of match values satisfies the increasing HR ordering. By contrast, if the conditions in Proposition 3(b) hold, we have $\partial\lambda^*/\partial c < 0$. If, in addition, $\partial p^*/\partial\lambda^* < 0$, the total derivative in (18) will also be positive; this occurs when the distribution of match values satisfies the decreasing HR ordering. When $\partial p^*/\partial\lambda^*$ and $\partial\lambda^*/\partial c$ have different signs, the impact of search costs on price via the reservation value is positive and via investment in quality negative. Which effect dominates is distribution dependent.

Part (a) of this Proposition gives examples of families of distributions for which the equilibrium price increases in search costs. For the Uniform II and Kumaraswamy II distributions, quality investment increases in search costs and because both distributions have a HR constant in λ the implication follows. Part (b) of Proposition 5 is perhaps the most surprising. For the Exponential I distribution, an increase in search costs results in a lower quality. But because such a distribution has a hazard rate that is constant in ε and decreasing in λ, price falls as search costs rise. Finally, in part (c) we give the case of the Exponential II distribution which has a HR constant in λ.

4 Efficiency

In this section, we ask whether the market provides too little or too much quality from a social welfare perspective. To address this question, we derive the social (first-best) optimum and compare it to the market equilibrium.

Social welfare is measured by the sum of expected consumer surplus and expected industry profit. If the investment level is λ^o, price is p^o and the planner dictates consumers to search with a reservation value $\hat{\varepsilon}^o$, then the expected consumer surplus, denoted by CS, can be written as:

$$CS(\hat{\varepsilon}^o, p^o, \lambda^o) = \frac{\int_{\hat{\varepsilon}^o} zf(z;\lambda^o)dz}{1 - F(\hat{\varepsilon};\lambda^o)} - \frac{c}{1 - F(\hat{\varepsilon}^o;\lambda^o)} - p^o.$$

The first term of this expression is the expected value of the match, the second is the expected search cost and the third is the price.

Industry profit is equal to:

$$\Pi(p^o, \lambda^o; \hat{\varepsilon}^o) = p^o - K(\lambda^o).$$

There is a unit mass of sellers. The revenue per firm is p^o because the number of consumers per firm is normalized to one and all consumers are served in equilibrium. The cost is just the investment cost because the marginal cost of production is normalized to zero.

Summing consumer surplus and industry profits we obtain an expression for welfare:

$$W(\hat{\varepsilon}^o, \lambda^o) = \frac{\int_{\hat{\varepsilon}^o} zf(z;\lambda^o)dz}{1 - F(\hat{\varepsilon}^o;\lambda^o)} - \frac{c}{1 - F(\hat{\varepsilon}^o;\lambda^o)} - K(\lambda^o). \tag{19}$$

Notice that in the welfare expression the price p^o cancels out. This is because the price is just a transfer between consumers and firms and has no bearing on aggregate surplus. Welfare is then the expected value of the match minus expected search and investment costs.

Taking the FOC of the social welfare expression in (19) with respect to $\hat{\varepsilon}^o$ gives:

$$\frac{\partial W}{\partial \hat{\varepsilon}^o} = -\hat{\varepsilon}^o f(\hat{\varepsilon}^o, \lambda^o)(1 - F(\hat{\varepsilon}^o, \lambda^o)) + f(\hat{\varepsilon}^o, \lambda^o)\left[\int_{\hat{\varepsilon}^o} zf(\hat{\varepsilon}^o, \lambda^o)dz - c\right] = 0, \tag{20}$$

Using integration by parts, this FOC can be rewritten as:

$$\int_{\hat{\varepsilon}^o} (1 - F(z;\lambda^o))dz - c = 0. \tag{21}$$

Comparing (21) to the market search rule (14), we observe that they are exactly identical. This implies that if the equilibrium has an efficient amount of investment, i.e. $\lambda^* = \lambda^o$, then search will also be efficient, i.e. $\hat{\varepsilon}^*(\lambda^*) = \hat{\varepsilon}^o(\lambda^o)$. Otherwise, inefficient investment will result in inefficient search.

Using (21), we can simplify the welfare expression in (19) to:

$$W(\hat{\varepsilon}^o, \lambda^o) = \hat{\varepsilon}^o(\lambda^o) - K(\lambda^o).$$

Thus, the socially optimal investment must maximize the reservation value of consumers minus investment cost. Taking the FOC with respect to investment gives:

$$\frac{\int_{\hat{\varepsilon}^o} \frac{\partial(1-F(z,\lambda^o))}{\partial \lambda^o} dz}{1 - F(\hat{\varepsilon}^o, \lambda^o)} - K'(\lambda^o) = 0, \tag{22}$$

where we have used the search rule in (21) to compute the derivative $d\varepsilon^o/d\lambda^o$.

The social optimum is given by the pair $(\hat{\varepsilon}^o, \lambda^o)$ that solves Eqs. (21) and (22). Comparing the FOC for the planner in (22) with that of the firms in (15), we can now study whether the market provides too much or too little incentives to invest in quality.

Proposition 6 *The market (over-) under-provides quality and consumers' reservation values are too (high) low if and only if*

$$\int_{\hat{\varepsilon}} \frac{\partial(1 - F(z, \lambda))}{\partial \lambda} dz (<) > p^* \frac{\partial(1 - F(\hat{\varepsilon}, \lambda))}{\partial \lambda} \tag{23}$$

Proof. See the Appendix. ■

The condition in Proposition 6 stems from a comparison of the social and the private incentives to invest in quality. The LHS of (23) represents the per-buyer marginal social gains from investment in quality. The RHS of (23) stands for the per-buyer private incentives to invest in quality. The social gains caused by an increase in quality equal the difference between the increase in the value of the match and the increase in the costs of search. The private gains equal the market value of the incremental rise in consumers' stopping probability. While the planner cares about the quality of the match between consumers and firms and search costs, the firms only care about consumers' stopping probability. The social planner and the firms thus have different valuations for quality increments, thereby creating a source of potential market failure.

Interestingly, the necessary and sufficient condition for over- or under-investment in Proposition 6 is related to the relationship between quality and the number of times consumers search in the market. In fact, because the equilibrium price is equal to the inverse hazard rate, the expressions in (7) and (23) are the same. Therefore, we can state without proof that:

Proposition 7 *The market under-provides (over-provides) quality if and only if a small increase in quality results in a higher (lower) number of searches.*

The intuition is that firms like consumers to stop search soon, while the planner, caring about match quality, prefers that they search more throughly. When a rise in quality leads to a higher (lower) number of searches, firms under-provide (over-provide) quality because the under-provision (over-provision) results in consumers stopping earlier. Because the number of searches increases in consumers' reservation value, combining Propositions 1 and 7 leads to the conclusion that when the market under-provides quality, consumers inspect too few products. When the market over-provides quality, the optimality of the number of searches is ambiguous though.

5 Conclusion

This paper has performed a positive and normative analysis of the market provision of quality in the work-horse model of consumer search for differentiated products. An increase in quality investment shifts up the distribution of match utilities offered by firms and makes consumers pickier. The typical number of products consumers inspect before settling, however, does not necessarily increase in quality. This is because, on the one hand, keeping consumers' reservation value fixed, the number of products consumers check goes down as with higher quality they find a suitable product faster; on the other hand, because consumers become more choosy as quality increases, they tend to discard more options while they inspect products. Either of these two effects may dominate. Intuitively, consumers will inspect fewer products when a rise in quality increases the upper tail of the distribution of match values less than other lower parts of the distribution. We expect this to occur when quality does not benefit more those consumers who derive high utility from the product. To mention an example drawn from mobile telephony, quality features such as robustness, battery performance and water resistance are likely to be more valued by the average buyer than by the true mobile phone fans who probably care more about features such as front-camera quality or AirDrop range. Likewise, in the car market, quality characteristics such as reliability and environmental performance are probably more valued by the average consumer than by the car lovers, who probably are more seduced by characteristics such as top speed, acceleration or quality sound system.

The analysis has also clarified how the incentives to invest in quality depend on search costs and how they relate to the social incentives. From a normative point of view, we have seen that the only inefficiency of the market equilibrium is due to the inadequacy of quality investment. We have also shown the existence of a close relationship between the intensity of search and the inefficiency of the market equilibrium. The market under-provides (over-provides) quality if and only if a rise in quality results in a higher (lower) number of searches. Returning to the examples above, our model predicts mobile phones' features such as robustness, battery performance and water resistance to be over-provided, while front-camera quality or AirDrop range to be under-provided. Likewise, in the car market, quality characteristics such as reliability and environmental performance are probably over-provided, while top speed, acceleration or quality sound system are under-provided. Our paper also carries a practical empirical implication. If the empirical researcher observes the number of searches in a market and there is enough quality variation in the data, the relationship between intensity of search and quality may readily be applied to test whether quality investment is excessive or insufficient from the point of view of welfare.

Appendix

Proof of Proposition 1. It remains to prove that the number of searches is constant in quality when quality enters additively and it increases in quality when quality enters multiplicatively.

For the additive case, ε is distributed on the interval $[\underline{\varepsilon}+\lambda, \overline{\varepsilon}+\lambda]$ according to distribution $F(\varepsilon-\lambda)$. Then, we have:

$$\frac{\int_{\hat{\varepsilon}}^{\overline{\varepsilon}(\lambda)} \frac{\partial(1-F(z,\lambda))}{\partial\lambda}dz}{1-F(\hat{\varepsilon},\lambda)} = \frac{\int_{\hat{\varepsilon}}^{\overline{\varepsilon}+\lambda} f(z-\lambda)dz}{1-F(\hat{\varepsilon}-\lambda)} = \frac{1-F(\hat{\varepsilon}-\lambda)}{1-F(\hat{\varepsilon}-\lambda)} = 1$$

and

$$\frac{\frac{\partial(1-F(\hat{\varepsilon},\lambda))}{\partial\lambda}}{f(\hat{\varepsilon},\lambda)} = \frac{\frac{\partial(1-F(\hat{\varepsilon}-\lambda))}{\partial\lambda}}{f(\hat{\varepsilon}-\lambda)} = \frac{f(\hat{\varepsilon}-\lambda)}{f(\hat{\varepsilon}-\lambda)} = 1.$$

This implies that the LHS and RHS of (8) are equal to one another and thus the number of searches is constant in λ.

For the multiplicative case, ε is distributed on the interval $[\underline{\varepsilon}(1+\lambda), \overline{\varepsilon}(1+\lambda)]$ according to distribution $F(\varepsilon/(1+\lambda))$. Then, we have:

$$\frac{\int_{\hat{\varepsilon}}^{\overline{\varepsilon}(\lambda)} \frac{\partial(1-F(z,\lambda))}{\partial\lambda}dz}{1-F(\hat{\varepsilon},\lambda)} = \frac{\int_{\hat{\varepsilon}}^{\overline{\varepsilon}(1+\lambda)} f\left(\frac{z}{1+\lambda}\right)\frac{z}{(1+\lambda)^2}dz}{1-F\left(\frac{\hat{\varepsilon}}{1+\lambda}\right)} = \frac{1}{1+\lambda}\left(\hat{\varepsilon}+\frac{\int_{\hat{\varepsilon}}^{\overline{\varepsilon}(1+\lambda)}\left(1-F\left(\frac{z}{1+\lambda}\right)\right)dz}{1-F\left(\frac{\hat{\varepsilon}}{1+\lambda}\right)}\right)$$

and

$$\frac{\frac{\partial(1-F(\hat{\varepsilon},\lambda))}{\partial\lambda}}{f(\hat{\varepsilon},\lambda)} = \frac{f\left(\frac{\hat{\varepsilon}}{1+\lambda}\right)\frac{\hat{\varepsilon}}{(1+\lambda)^2}}{\frac{1}{1+\lambda}f\left(\frac{\hat{\varepsilon}}{1+\lambda}\right)} = \frac{\hat{\varepsilon}}{1+\lambda},$$

where in the first line we have integrated by parts. This implies that the LHS of (8) is greater than the RHS. Hence, the number of searches increases in λ. ■

Proof of Proposition 2.

After integration by parts, we can more conveniently rewrite the MRL function evaluated at the reservation value as follows:

$$MRL(\hat{\varepsilon};\lambda) = \frac{\int_{\hat{\varepsilon}}(1-F(z;\lambda))dz}{1-F(\hat{\varepsilon};\lambda)}.$$

Because the random variable ε satisfies the decreasing MRL ordering, the MRL is weakly decreasing in λ. This implies that the first-order derivative of $MRL(\hat{\varepsilon};\lambda)$ with respect to λ is non-positive. This holds if and only if:

$$(1-F(\hat{\varepsilon},\lambda))\int_{\hat{\varepsilon}}\frac{\partial(1-F(z,\lambda))}{\partial\lambda}dz \leq \frac{\partial(1-F(\hat{\varepsilon},\lambda))}{\partial\lambda}\int_{\hat{\varepsilon}}(1-F(z,\lambda))dz.$$

Moreover, because ε has a log-concave failure rate, its MRL decreases in ε (see e.g. Theorem 6 in Bagnoli and Bergstrom, 2005). This means that the first-order derivative of $MRL(\hat{\varepsilon};\lambda)$ with respect to $\hat{\varepsilon}$ is negative, which implies:

$$\int_{\hat{\varepsilon}}(1-F(z,\lambda))dz \leq \frac{(1-F(\hat{\varepsilon};\lambda))^2}{f(\hat{\varepsilon};\lambda)}$$

Combining the two previous inequalities gives:

$$\int_{\hat{\varepsilon}} \frac{\partial(1-F(z,\lambda))}{\partial\lambda} dz \leq \frac{\partial(1-F(\hat{\varepsilon},\lambda))}{\partial\lambda} \frac{(1-F(\hat{\varepsilon};\lambda))}{f(\hat{\varepsilon};\lambda)},$$

which is the same as condition (8). As a result, a small increase in λ weakly decreases the number of products consumers inspect. ■

Proof of Lemma 1.

Consider the function of λ defined by the RHS of (13). At $\lambda = 0$, this function is strictly positive because $K'(0) = 0$ and we have assumed that $\frac{\partial(1-F(\hat{\varepsilon},0))}{\partial\lambda} > 0$. At $\lambda = 1$, this function is strictly negative because $K'(1) = \infty$ and we have assumed that $\frac{\partial(1-F(\hat{\varepsilon},1))}{\partial\lambda}$ is finite. Therefore, the function of λ defined by the RHS of (13) crosses at least a single time the horizontal axis so there is at least one equilibrium candidate λ^*. Plugging an equilibrium candidate λ^* in (12) gives the corresponding equilibrium candidate p^*. Unicity of a firm equilibrium candidate (p^*, λ^*) obtains if the function of λ defined by the RHS of (13) is monotone decreasing in λ. Taking the derivative with respect to λ of the RHS of (13) gives the condition:

$$\frac{1}{f^2}\left(\frac{\partial^2(1-F)}{\partial\lambda^2} f - \frac{\partial(1-F)}{\partial\lambda}\frac{\partial f}{\partial\lambda}\right) - K''(\lambda) < 0.$$

■

The Hessian matrix.

For a shorter notation, let us express the payoff of a firm i as $\pi(p_i, \lambda_i) = p_i D(p_i, \lambda_i) - K(\lambda_i)$ where $D(p_i, \lambda_i) \equiv S(\lambda^*)(1 - F(\hat{\varepsilon} - p^* + p_i; \lambda_i)$. The Hessian matrix corresponding to the payoff $\pi(p_i, \lambda_i)$ is:

$$H = \begin{bmatrix} 2D'_{p_i} + p_i D''_{p_i p_i} & D'_{\lambda_i} + p_i D''_{p_i \lambda_i} \\ D'_{\lambda_i} + p_i D''_{p_i \lambda_i} & p_i D''_{\lambda_i \lambda_i} - K'' \end{bmatrix} \tag{24}$$

where $D'_{p_i} = \frac{\partial D}{\partial p_i}$, $D''_{p_i p_i} = \frac{\partial^2 D}{\partial p_i^2}$, $D''_{p_i \lambda_i} = \frac{\partial^2 D}{\partial p_i \partial \lambda_i}$ and $D''_{\lambda_i \lambda_i} = \frac{\partial^2 D}{\partial \lambda_i^2}$. The payoff $\pi(p_i, \lambda_i)$ is strictly concave in both p_i and λ_i when the Hessian matrix is definite negative. With $D''_{p_i p_i} \leq 0$, the first leading principal minor of the matrix H is clearly negative. Assuming $D''_{p_i p_i} \leq 0$, the second leading principal minor of the matrix H is positive when

$$K'' > p_i D''_{\lambda_i \lambda_i} - \frac{\left(D'_{\lambda_i} + p_i D''_{p_i \lambda_i}\right)^2}{2D'_{p_i} + p_i D''_{p_i p_i}}.$$

These two conditions imply that a candidate firm equilibrium is indeed a firm equilibrium when demand is strictly concave in p_i and the cost of investment function is sufficiently convex. ■

Proof of Proposition 3.

Part (a). First, note that by the implicit function theorem, (14) defines a relation $\hat{\varepsilon}_1(\lambda)$. At $\lambda = 0$, $\hat{\varepsilon}_1(0)$ is strictly positive because $F(\varepsilon, 0)$ is a well-defined distribution. Moreover, as explained above, $\hat{\varepsilon}_1(\lambda)$ is increasing in λ.

Second, note that, when K is sufficiently convex, a solution to (15) exists (cf. Lemma 1). This solution defines a relation $\hat{\varepsilon}_2(\lambda^*)$, which, under assumption (a), decreases in λ. To see this, we apply the implicit function theorem to equation (15):

$$\frac{\partial \hat{\varepsilon}_2}{\partial \lambda} = -\frac{\frac{1}{f^2}\left(\frac{\partial^2(1-F)}{\partial \lambda^2} f - \frac{\partial(1-F)}{\partial \lambda}\frac{\partial f}{\partial \lambda}\right) - K''(\lambda)}{\frac{1}{f^2}\left(-\frac{\partial f}{\partial \lambda} f - \frac{\partial(1-F)}{\partial \lambda}\frac{\partial f}{\partial \hat{\varepsilon}}\right)}.$$

Next we note that under assumption (a), the denominator of $\partial\hat{\varepsilon}_2/\partial\lambda$ is negative while the numerator, provided that K is sufficiently convex, is negative. Thus the implicit function $\hat{\varepsilon}_2(\lambda)$ decreases in λ.

Moreover, by assumption $\hat{\varepsilon}_2(0) > \hat{\varepsilon}_1(0)$, which implies that $\hat{\varepsilon}_1(\lambda^*)$ and $\hat{\varepsilon}_2(\lambda^*)$ cross once and only once, guaranteeing a unique solution.

Part (b). Under assumption (b), the denominator of $\partial\hat{\varepsilon}_2/\partial\lambda^*$ is positive and by sufficient convexity of K the numerator is negative. Therefore, $\hat{\varepsilon}_2(\lambda^*)$ increases in λ^*. Further, by assumption $\hat{\varepsilon}_2(0) < \hat{\varepsilon}_1(0)$. Furthermore, under sufficient convexity of K, the assumptions ensure that $\hat{\varepsilon}_2(\lambda)$ is sufficiently increasing, which implies that $\frac{\partial \hat{\varepsilon}_1}{\partial \lambda^*} < \frac{\partial \hat{\varepsilon}_2}{\partial \lambda^*}$. Therefore $\hat{\varepsilon}_1(\lambda^*)$ and $\hat{\varepsilon}_2(\lambda^*)$ surely cross one another only once, which guarantees the existence of a unique solution. ■

Proof of Proposition 6.

Because, consumers search efficiently for a fixed λ, to prove the result we only need to compare the FOCs of the planner and the firms, given by Eqs. (15) and (22). Taking the difference of the two FOCs gives:

$$\begin{aligned}\frac{\partial W}{\partial \lambda} - \frac{\partial \pi}{\partial \lambda} =& \frac{\int_{\hat{\varepsilon}} \frac{\partial(1-F(z,\lambda))}{\partial \lambda} dz}{1-F(\hat{\varepsilon},\lambda)} - \frac{\frac{\partial(1-F(\hat{\varepsilon},\lambda))}{\partial \lambda}}{f(\hat{\varepsilon},\lambda)} \\ =& \frac{1}{1-F(\hat{\varepsilon},\lambda)}\left[\int_{\hat{\varepsilon}} \frac{\partial(1-F(z,\lambda))}{\partial \lambda} dz - p^* \frac{\partial(1-F(\hat{\varepsilon},\lambda))}{\partial \lambda}\right] \qquad (25)\end{aligned}$$

Let the solution to (22) be $\hat{\varepsilon}_3(\lambda)$. When (25) is positive, the solution to (22) is smaller than the solution to (15), which we have denoted up until now by $\hat{\varepsilon}_2(\lambda)$. As a result, the crossing point between $\hat{\varepsilon}_1(\lambda)$ and $\hat{\varepsilon}_2(\lambda)$, which gives the market equilibrium $(\hat{\varepsilon}, \lambda^*)$ will be below the crossing point between $\hat{\varepsilon}_1(\lambda)$ and $\hat{\varepsilon}_3(\lambda)$, which gives the social optimum $(\hat{\varepsilon}^o, \lambda^o)$. This implies under-investment. Becasue $\hat{\varepsilon}_1(\lambda)$, consumers are too little picky. When (25) is negative, we have over-investment and too picky consumers. ■

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